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The Aerodynamical Equations of the Propeller Blade Elements.

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Summary.

The air flow and the air force created by all elements of the propeller blades lying in a ring located between two concentric circles around the propeller axis are independent of what happens in other rings. The aerodynamic equations of such blade elements are established and their application is discussed.

Since the air forces of well-designed propellers are uniformly distributed along the blades, the blade elements in each concentric ring around the propeller axis have properties practically independent of those in other rings. Hence the design or the analysis of a propeller has to be performed by designing or analyzing consecutively the elements of a number of such rings. I proceed immediately to the discussion of this problem.

One such short piece of the blade is generally considered as a wing moving on a spiral path. The angle of each blade element may always be measured from its direction of zero lift. There are two important angles: The angle ϵ of inclination to the propeller plane and the angle β of attack relative to the air. The air force of the blade element has one component in the direction of the relative motion and another one at right angle to it. The former is due to the viscosity of the air and will be considered later. It is small in general and does not materially alter the character of the air flow, but it can by no means be neglected. It may be called drag, and the other force may be called lift. By dividing the lift by the area of the blade element and by the dynamical pressure of the relative velocity between blade element and air, a lift coefficient C_L may be defined. The lift coefficient thus obtained obeys the laws established in the ordinary wing theory for infinite aspect ratio. In particular, it is never very different from $2\pi\beta$.

The lift of the blade element, the relative velocity between it and the air, and the absolute velocity of the blade elements may be divided into components parallel and at right angles to the propeller axis. The components of the relative velocity may be written u and v, and those of the absolute velocity, U and V, where v and V are parallel to the axis. It may be assumed that V at the same time is the velocity of flight. The two components of the lift may be called thrust T and tangential force P. Let δ finally be the angle between the direction of the lift and the axis, so that

$$T = L\cos\delta, \qquad P = L\sin\delta, \qquad v/u = \tan\delta .$$

The angle of inclination of the blade element to the plane of the propeller is then

$$\epsilon = \delta + \beta.$$

The problem is to deduce the values of U and V if δ, C_L, u, and v are given. This is done by the application of the momentum theorem. The mass of air to which momentum is imparted is the mass of air passing the ring of radius r and breadth dr with the velocity v. Hence it is per unit of time

$$2 \pi r \, dr \, v . \rho .$$

Momentum is imparted by both the thrust and the tangential force, and it has to be assumed that half the total gain in velocity has been made when the air passes the propeller. Therefore, half this gain in velocity is

$$v \frac{i\,t}{2 \pi r} \frac{C_L}{4} \cdot \frac{1}{\sin \delta}$$ in the tangential direction, and

$$v \frac{i\,t}{2 \pi r} \frac{C_L}{4} \cdot \frac{\cos \delta}{\sin^2 \delta}$$ in the axial direction, where i denotes the number of blades and t the breadth.

Hence, writing m for

$$\frac{i\,t}{2 \pi r} \frac{C_L}{4}.$$

(1) $$V = v(1 - m \cos \delta / \sin^2 \delta) .$$

(2) $$U = u(1 + m/\sin \delta).$$

From which it follows that

(3) $$\frac{V}{U} = \tan \delta \, \frac{1 - m \cos \delta / \sin^2 \delta}{1 + m/\sin \delta} :$$

The propeller coefficients, defined by

$$C_T = \frac{T}{2\,\pi\,r\,dr\,V^2\,\frac{\rho}{2}} \quad \text{and } C'_Q = \frac{P}{2\,\pi\,r\,dr\,V^2\,\frac{\rho}{2}}$$

therefore is

$$(4) \qquad C_T = \frac{4\,m\,\cos\delta}{(1 - m\,\cos\delta/\sin^2\delta)^2}\ ;\ C'_Q = C_T\,\tan\delta .$$

There remains still the drag to be considered. Its influence on the thrust and on the acceleration can be neglected. The coefficient C'_Q is increased by

$$C''_Q = \frac{4\,m\,\cos\,C_D/C_L}{(1 - m\,\cos\,\delta/\sin^2\delta)^2}$$

as follows from the consideration that the tangential component of the drag is $D \cos\delta$ and by applying the same conclusions as before. Hence the complete coefficient of the tangential force is

$$(5) \qquad C_Q = \frac{4\,m\,\cos\,\delta}{(1 - m\,\cos\,\delta/\sin^2\delta)}\ (\tan\delta\ +\ C_D/C_L).$$

The equations thus obtained are sufficient for the design and the analysis of the blade element, but they are not yet in such a form that they can be used conveniently. For this purpose the equations have to be transformed. For purposes of design, generally C_T, C_L and V/U are given, and also the number of blades. From C_L, β and C_D can be derived directly from the knowledge of the wing section and are to be considered as known too. The three unknown quantities to be determined are the breadth of the blade element, t, its angle of inclination, ϵ, and C_Q, the coefficient of the tangential force. These can easily be found if the two quantities δ and m are known. They are to be determined first. From equation (3):

$$(6) \qquad m = \sin\delta \frac{\tan\delta - V/U}{1 + V/U} .$$

This may be substituted in equation (4)

$$C_T = \frac{4 \sin\delta \cos\delta \dfrac{\tan\delta - V/U}{1 + V/U}}{\left(1 - \dfrac{\tan\delta - V/U}{\tan\delta (1 + V/U)}\right)^2}$$

or, otherwise written

$$(7) \quad \tan\delta = V/U + \tan\delta (1 + V/U) \left\{1 + \frac{2\sin^2\delta}{C_T} - \sqrt{\left(1 + \frac{2\sin^2\delta}{C_T}\right)^2 - 1}\right\}.$$

The negative value of the square root has to be taken, because otherwise, if C_T is small, the second term on the right hand side becomes very great.

This equation now can be used for the determination of $\tan\delta$. It is true that the right hand side contains the unknown quantity too, but it contains it in such a way that its value is not greatly affected by a small error in $\tan\delta$. The procedure is therefore to estimate $\tan\delta$ at first. It is slightly greater than V/U, say 10% for high velocities of flight and 20% for low speeds. Hence 1.1 to 1.2 V/U is to be substituted for $\tan\delta$

in the right hand side of the equation, thus giving a second improved value for $\tan \delta$. Often this improved value will be close enough to the first estimation and to the final result. If not, this improved second approximation may be substituted in the right hand side of equation (7) giving a third improved approximation, and so on.

After the determination of δ, m is found by substituting δ in equation (6). Finally,

$$(8) \qquad t = \frac{4m}{C_L} \, \frac{2 \pi r}{i}$$

$\epsilon = \beta + \delta$ and C_Q can be found by substituting m and δ in the right side of equation (5).

Thus all quantities needed for purposes of design are determined.

The analysis of a given blade element, that is, the calculation of the air force if the width of the blade element, t, its inclination, ϵ, and its velocity, U/V, are given, is performed similarly. The same equations are to be applied of course. It may be assumed that U/V, ϵ, and $\frac{i\ t}{2 \pi r}$ are given. In addition, the relation between β and C_L must be known and may be assumed to be

$$C_L = 2 \pi \beta .$$

C_T and C_Q are to be determined. In order to be able to determine them, δ and m are calculated first.

The three equations for the determination of the unknown quantities are now

$$m = \frac{i\ t}{2\ \pi\ r}\ \frac{C_L}{4}$$

$$\delta = \epsilon - \frac{C_L}{2\pi}$$

$$V/U = \tan\delta\ \frac{1 - m\cos\delta/\sin^2\delta}{1 + m/\sin\delta}$$

with the three unknown quantities m, δ, and C_L.

The first equation can be written

$$C_L = 4\ m\ \frac{2\ \pi\ r}{i\ t}$$

and this can be substituted in the second equation, giving

$$\delta = \epsilon - \frac{2m}{\pi}\ \frac{2\ \pi\ r}{i\ t}.$$

Apply the function tan on both sides, taking into consideration that the last term is small

$$\tan\delta = \frac{\tan\epsilon - \frac{2m}{\pi}\ \frac{2\ \pi\ r}{i\ t}}{1 + \frac{2m}{\pi}\ \frac{2\ \pi\ r}{i\ t}\tan\epsilon}$$

or transformed

$$m = \frac{\tan\epsilon - \tan\delta}{\frac{2}{\pi}\ \frac{2\ \pi\ r}{i\ t}\ (1 + \tan\epsilon\ \tan\delta)}$$

Now the value of m, as derived from the third equation, is given by equation (6). The two expressions have to be equal and hence they give an equation which contains only the one unknown quantity $\tan\delta$. The resulting equation can be written

$$(9) \quad \tan\delta = \frac{\tan\epsilon \quad (1 + V/U)\ \sin\delta + V/U\ \frac{4r}{i\,t}\ (\tan\epsilon \quad \tan\delta \ + 1)}{(1 + V/U)\ \sin\delta + \quad \frac{4r}{i\,t}\ (\tan\epsilon \quad \tan\delta \ + 1)} .$$

This equation can be used for the determination of $\tan\delta$ in the same way as was equation (6). $\tan\delta$ has to be assumed to have a value between $\tan\epsilon$ and V/U.

If $\tan\delta$ is found, C_L is quickly calculated by means of

$$(10) \qquad C_L = 2\pi \quad (\epsilon - \delta)$$

and

$$(11) \qquad m = \frac{i\ t}{2\ \pi\ r}\ \frac{C_L}{4} = \frac{i\ t}{2\ \pi\ r}\ \frac{(\epsilon - \delta)\ \pi}{2} .$$

and thus all quantities occurring in equations (4) and (5) are known and, being substituted, will give the values of C_T and C_Q.

I wish to add some remarks referring to the actual solution of equations (7) and (9). The determination of $\tan\delta$ can be made in less than one minute if the following rules are observed.

In both equations occur $\tan\delta$ and $\sin\delta$ and it would take too much time to look for $\sin\delta$ in a table after $\tan\delta$ is given or assumed. The sine can be determined from the tangent much more quickly by means of the slide rule. For

$$\sin\delta = \frac{\tan\delta}{\sqrt{1 + \tan^2\delta}} .$$

This can be solved by one move of the runner and the rule. The reader is requested to take his slide rule and to compute the following example in order to convince himself how much more quickly he can perform the calculation than I can explain it. (See figure).

Suppose, for example, that $\tan\delta = .40$. Place the runner on the graduation .40 of the lower scale D. Then the runner indicates .16 on the upper scale A. Add 1 to this: $1 + .16 = 1.16$ and move the rule so that its upper scale B shows this 1.16 under the mark of the runner. Then the left end of the runner gives $\sin\delta = .372$ on the lower scale D and $\sin^2\delta = .138$ on the upper scale A.

In the solution of equation (7), the following expressions are to be computed, in the order given, by means of the slide rule: Given V/U, C_T, and $\tan\delta$, approximately

$$(a) = \frac{2\sin^2\delta}{C_T}$$

$$(b) = 1 + (a)$$

$$(c) = \sqrt{(b)^2 - 1}$$

$$(d) = (b) - (c)$$

$$(e) = \tan\delta\ (1 + V/U) \cdot (d)$$

$$\mathrm{tg}\,\delta = V/U + (e)$$

If (b) is considerably greater than 1, (d) has the value $\dfrac{1}{2\,(b)}$.

For the solution of equation (9): Given V/U, $\tan\epsilon$, $\dfrac{4\,r}{i\,t}$ and $\tan\delta$ approximately (it lying between V/U and $\tan\epsilon$).

$$(a) = 1 + \tan\epsilon \ \tan\delta$$

$$(b) = (1 + V/U)\ \sin\delta \qquad (c) = \tan\epsilon\ (b)$$

$$(d) = \frac{4\ r}{i\ t}\ (a) \qquad (e) = V/U\ (d)$$

$$(f) = (b) + (d) \qquad (g) = (c) + (e)$$

$$\tan\delta = \frac{(g)}{(f)} .$$